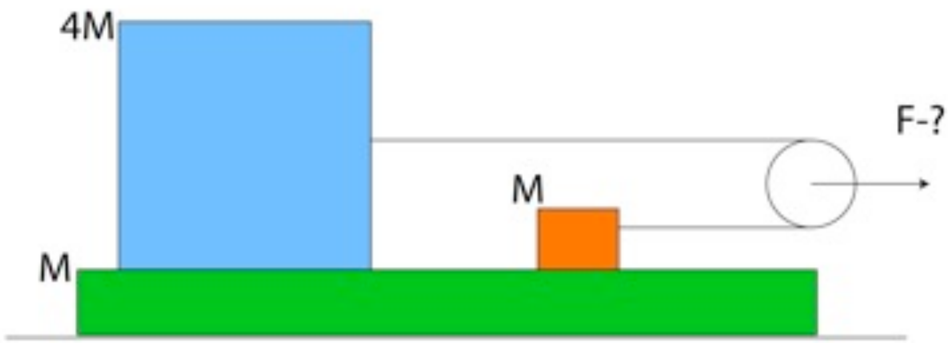
In the diagram, a platform of mass M is placed on a horizontal frictionless surface. Two blocks of masses $4M$ and M are placed on the platform. For both blocks , the coefficients of static friction with the platform equal 0.16 and the coefficients of kinetic friction equal 0.10. the blocks are connected by a light string through a light pulley, which is acted upon by an unknown horizontal force F. If the acceleration of the platform is $0.2g$, find the value of F and the acceleration of each block.



With a little thought you will conclude that it is impossible for the platform to have the given acceleration if both blocks are either stuck to the platform or both sliding with respect to the platform. Since the smaller block will have a lower maximum allowable frictional force than the larger block it must be sliding, while the larger is fixed to the platform.

The second thing to realize is that for the larger block, the static frictional force between it and the platform will be somewhere between zero and its maximum value $4Mg\mu_s$.

Step 1: Let's consider the forces acting on the platform.

There are two forces acting to accelerate it to the right: a force due to the blue block (F_B) and a force due to the orange block ($Mg\mu_k$). Together they must equal M times the acceleration of the green platform (a_G).

$$F_B + Mg\mu_k = Ma_G \qquad \text{Eq. 1}$$

Step 2: Let us consider now the forces on the blue block. From the 3rd law there is $-F_B$, and there is the tension in the string T

$$T - F_B = 4Ma_G \qquad \text{Eq. 2}$$

Note that its acceleration must be the same as the green platform since it is not sliding with respect to it.

Step 3: And finally, the forces on the orange block. We have the tension acting to the right and the kinetic frictional force acting to the left:

$$T - Mg\mu_k = Ma_O \qquad \text{Eq. 3}$$

where a_O is the acceleration of the orange block.

These are all the kinematic equations we can set up. So let us proceed to solve them.

Since we do not know F_B, let's eliminate it by substituting Eq. 1 into Eq. 2:

$$T - \left[Ma_G - Mg\mu_k \right] = 4Ma_G \, .$$

Simplifying:

$$T = 5Ma_G - Mg\mu_k \, .$$

Eliminate T by substituting into Eq. 3:

$$5Ma_G - Mg\mu_k - Mg\mu_k = Ma_O \, .$$

Canceling the Ms and solving for the acceleration of the orange block:

$$a_O = 5a_G - 2g\mu_k \, .$$

Recall that $a_G = 0.2g$ and $\mu_k = 0.1$, we find that:

$$a_O = g\left(5\left(0.02\right) - 2\left(0.1\right)\right) = 0.8g \, .$$

So, while the platform and the large blue block have an acceleration of 0.2g, the orange block has a larger acceleration of 0.8g.

Substituting the above back into Eq. 3

$$T = Mg\left(0.8\right) + Mg\left(0.1\right) = 0.9Mg$$

Since the pulley mass is negligible $F = 2T$ so

$$F = 1.8Mg \, .$$

We can also check that F_B is less that the maximum static frictional force. From Eq. 2:

$$F_B = T - 4Mg\left(0.2\right) = Mg\left[0.9 - 4\left(0.2\right)\right] = 0.1Mg \, .$$

which is less than the maximum supportable static frictional force between the blue block and the platform:

$$Ff_{Max} = 4Mg\left(0.16\right) = 0.64Mg \, .$$

Quick and Dirty Solution:

Since the large block is not moving with respect to the platform, we can pretend that it is a part of the platform itself and then we will only have two equations, one for the platform/large block system and one for the small block:

$$Mg\mu_k + T = 5Ma_p$$

$$T - Mg\mu_k = Ma_O$$

If we eliminate tension:

$$Mg\mu_k + \left(Ma_O + Mg\mu_k\right) = 5Ma_p \Rightarrow a_O = 5a_p - 2g\mu_k$$

Which is the same as what we obtained above ($a_p = a_G$). Plug back in to find the tension and you are done.